of $Q^m_{n-1/2}(s)$ for $m = 5(1)10$, $s = 1.1(0.1)10$, and n varying from 0 through consecutive integers to a value ranging from 35 to 160 for which the value of the function relative to that when n is zero is less than 10^{-21}.

Also, as in the first report, this is immediately followed by the tabulation of the same function to the same precision and for the same orders, m, but for arguments $s = \cosh \eta$, where $\eta = 0.1(0.1)3$. Here the upper limit for the degree, n, varies from 34 to 450.

No explanatory text accompanies these tables; accordingly, the user should consult the first report for a mathematical discussion of these functions and the various methods used in the preparation of the tables, as well as for additional references.

J. W. W.

1. Henry E. Fettis & James C. Caslin, *Tables of Toroidal Harmonics*, I: *Orders* 0–5, *All Significant Degrees*, Report ARL 69-0025, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, February 1969. (See *Math Comp.*, v. 24, 1970, p. 489, RMT 36.)

71[7].—K. A. Karpov & E. A. Čhistova, *Tablit͡sy Funkt͡siĭ Vebera*, Tom III (*Tables of Weber Functions*, v. III), Computing Center, Acad. Sci. USSR, Moscow, 1968, xxiv + 215 pp., 27 cm. Price 2.05 rubles.

Weber functions are defined as solutions of the differential equation

$$\frac{d^2y}{dz^2} + (p + \tfrac{1}{2} - \tfrac{1}{4}z^2)y = 0. \tag{1}$$

Whittaker's solution $D_p(z)$ of (1) may be defined by the initial values

$$D_p(0) = \frac{2^{p/2}\sqrt{\pi}}{\Gamma\left(\frac{1-p}{2}\right)}, \qquad D_p'(0) = -\frac{2^{p/2}(2\pi)^{1/2}}{\Gamma\left(-\frac{p}{2}\right)}$$

and is characterized by the asymptotic behavior

$$D_p(z) \sim e^{-z^2/4}z^p \quad \text{as } z \to \infty \text{ in } |\arg z| < \pi/2.$$

If p is not an integer, then $D_p(z)$, $D_p(-z)$ and $D_{-p-1}(iz)$, $D_{-p-1}(-iz)$ are pairs of linearly independent solutions of (1).

The function $D_p(z)$ for real p and $z = x(1 + i)$, x real, has been tabulated in two earlier volumes [1], [2]. The present volume tabulates $D_p(z)$ for z real and purely imaginary, and p real, and completes the tabulation of Weber functions undertaken by the Computing Center of the U.S.S.R. Academy of Sciences.

There are three principal tables in the present volume. The first gives $D_p(x)$ for $0 \leqq x < \infty$; the second, $\exp(-x^2/4)D_p(x)$ for $-\infty < x \leqq 0$; and the third, the real and imaginary parts of $\exp(-x^2/4)D_p(ix)$ for $0 \leqq x < \infty$. The tabular interval in x is 0.01 for $|x| \leqq 5$, and 0.001, or 0.0001, in $y = 1/x$ for $|x| > 5$. The range in p is $-1(0.1)1$ throughout, but can be extended with the aid of recurrence relations. All tabular entries are given to 7D, if less than 1 in absolute value; otherwise they are given to 8S.

The introduction contains detailed comments on interpolation and on methods for extending the tabular range. Many worked examples are included, as well as auxiliary tables. Also included are eight graphs and three reliefs illustrating the behavior of the functions tabulated.

W. G.

1. I. E. KIREEVA & K. A. KARPOV, *Tablit͡sy Funkt͡siĭ Vebera*, v. I, Computing Center, Acad. Sci. USSR, Moscow, 1959. Edition in English, Pergamon Press, New York, 1961. (See *Math. Comp.*, v. 16, 1962, pp. 384–387, RMT 38.)

2. K. A. KARPOV & E. A. C͡HISTOVA, *Tablit͡sy Funkt͡siĭ Vebera*, v. II, Computing Center, Acad. Sci. USSR, Moscow, 1964.

72[7].—DZ͡H. C͡H. P. MILLER, *Tablit͡sy Funktsiĭ Vebera* (J. C. P. MILLER, *Tables of Weber Functions*), Computing Center, Acad. Sci. USSR, Moscow, 1968, cxvi + 143 pp., 27 cm. Price 1.69 rubles.

Weber functions (or parabolic cylinder functions) in Whittaker's standardization are solutions of

$$\frac{d^2y}{dx^2} - (\tfrac{1}{4}x^2 + a)y = 0 \tag{1}$$

or solutions of

$$\frac{d^2y}{dx^2} + (\tfrac{1}{4}x^2 - a)y = 0. \tag{2}$$

Although the second equation may be obtained from the first by simultaneous replacement of a by $-ia$ and x by $xe^{i\pi/4}$, it is convenient to consider each equation separately when dealing with the real-variable theory of these equations.

Both equations arise naturally in the solution of Helmholtz's equation upon separation of variables in parabolic cylinder coordinates. They also occur in the asymptotic theory of second-order differential equations with turning points. For special values of a the solutions of (1) are related to the normal error function and its repeated integrals and derivatives.

One owes to J. C. P. Miller [1] a thorough mathematical treatment of Weber functions, covering both equations (1) and (2), and the first attempt at systematic tabulation of the solutions of (2) for real x and a.

The volume under review is a Russian translation of [1] by M. K. Kerimov. All tables and mathematical formulas appear to have been reproduced photographically from the original.

A supplementary section added by the translator contains additional material on Weber functions, mostly of recent origin. In particular, one finds an account of the asymptotic theory of these functions and their zeros as developed by Olver in the late 1950's miscellaneous results such as integral representations, limit relations, addition theorems, infinite series and integrals involving Weber functions, as well as a survey of recent tables and computer programs. The bibliography of cited references contains some 200 items.

W. G.